数学史研究

（通巻 58 号）

1973 年 7 月～9月

目次

和算紹介論文集（英文）

藤沢利喜太郎 .......................................................... 和算ノート ……1

菊池大蔵 .............................................................. 円理関係論文 ……18

①円の面積 ②π ③π²

④円周の弧背術 ⑤円の弧背術

解説（下平和夫） ....................................................... 59

編集後記 .............................................................. 60

日本数学史学会
14.

Note on the Mathematics of the Old Japanese School.

(Compte Rendu du 2. Congrès International des Mathématiciens, Paris, 1900, pp. 379-393.)

The present short note is devoted to a subject which is entirely obsolete nowadays, and which appears to have only historical interest. It has been hastily written during my voyage, my coming to Paris to join this congress having been only settled definitively about a week before I left Tokio. Some of the works which I might have consulted were not accessible to me, and I had often to appeal to my memory alone. Owing to this circumstance and no less to the nature of the subject with which it deals, the present communication is vague in many respects. I crave your indulgence for this and many other shortcomings, which it may not be necessary to enumerate in this place.

There are circumstances which seem to suggest that, the bare fact that there is such a thing as the mathematics of the old Japanese school may not be altogether unknown to at least a few of the western mathematicians. To avoid misunderstanding, may I be allowed to say that I myself do not belong to this school. It might appear to be rather curious that I should speak upon a subject with which I am not familiar. To justify myself in this respect, may I be permitted to make a few remarks on the almost insurmountable difficulties which seem to accompany the study of this subject in the light of modern mathematics.

The nomenclature and the notations are as clumsy as they are awkward, and are likely to be repulsive to any one who is accustomed to those of the modern mathematics. Add to this, that there is a
NOTE ON THE MATHEMATICS OF THE OLD JAPANESE SCHOOL.

considerable number of minor or branch schools, into which this old
school has been subdivided in the course of time, each of them fol-
lowing notations which differ to a certain extent from one another.
In this connection, it may be observed that many of the difficulties
which one has to encounter in trying to make himself familiar with
the present subject, may be traced to the various controversies and
even jealousies which appear to have existed among these minor
schools.

There is an innumerable number of books relating to the subject,
of which a large number is not printed but preserved by transcrip-
tion. Since some time, the imperial university of Tokio has been collecting
books, manuscripts, etc., relating to the mathematics of the old Japa-
nese school. The collection now amounts to more than two thousand
volumes, but it seems to be only a small fraction of what must be
collected in order to have any claim to completeness. No doubt a
large number of these are devoted to subjects which are entirely
elementary; but it so happens that things which seem to be original
and valuable are mixed up with the calculations belonging to ele-
mentary arithmetic. On this account, one who would like to make him-
self master of the subject must go, with all the disadvantages of
varied notations, lack of systematisation and classification, etc.,
through almost all the innumerable books, in order not to miss
things which are really important. To start with, he knows that the
complete study of the subject is a life work, or even more, for him,
with the prospect that a large part of his work will prove to be a
waste labour. It is needless to say that this is by no means a fas-
cinating work; at the same time the difficulties which I have been
mentioning, and which may fairly be compared with those attending
the study of Egyptian hieroglyphs, are the difficulties which a histori-
an must be prepared to meet with. I hope to be able to find some
among the young men who are studying mathematics under my
direction, whom I could persuade to devote their lives to the profound
study of the subject of which the present note gives only a superficial
account. It is in view of having such men that the Imperial Univer-
sity of Tokio has been collecting the literature relating to this subject,
as before said.

In this connection, a new impetus was given by my friend and
former colleague Dr. Kikuchi. He has translated some of the meth-
ods of finding the value of \( \pi \), discovered by men of the old school,
from the obsolete language of the original into the intelligible lan-
guage of modern mathematics. His papers are contained in the
recent volumes and, I believe, also in the forthcoming volumes of the
proceedings of the Tokio Mathematical and Physical Society, to which
I have to refer for further particulars.

Relating more especially to those parts of the mathematics of the
old Japanese school which promise to have the chance of having
permanent value and interest, the following difficulties are to be met
with. Very often results only are given, and not seldom it is difficult
to trace the steps by which they might have been obtained. I can
not express myself better than by recalling the well known incident
connected with the publication of Fermat's theorems. Again it was
not customary with a mathematician of this school to publish what
he had discovered. He would keep it to himself and only transmit
it to posterity by telling it to a select few of his disciples under oath
that they in their turn shall follow the example set by their master.
Again what may be called problem-challenge or perhaps mathe-
tatical tournament--I mean something like the challenge of the problem
of the brachistochrone by a Bernoulli addressed to Newton, Leibniz
and Marquis de l'Hospital—has been constantly going on among the
various minor schools mentioned above. All these circumstances, to
which I could add some more of analogous nature, had the aggregate
effect of making the mathematics of this school extremely unintelli-
gible and the course of its development difficult to follow. That there
is a peculiar kind of mathematics which had its origin in Japan and
its secluded development free from external influences, is a fact; but
how far this mathematics has developed itself on the scale of the pro-
gress of modern mathematics, is a problem which, so far as I am
aware of, has not hitherto been solved,

Before concluding these preliminary remarks, it is only just that I
should mention that a few years ago a book entitled the history of
the mathematics of the old Japanese school was published by my friend Mr. T. Endo. The compilation of this work cost its author some sixteen years of arduous labour and undaunted diligence. He seems to have spared neither time nor pain in order to make the history as complete as possible, and I have no doubt that it will serve as a useful guide to all the future students of this peculiar mathematics. I, myself, owe a great deal to this work for the little I know of this mathematics and I take this opportunity to tender my most cordial thanks to its author. Only the fear that I might be misunderstood as agreeing with the author in those parts of his work which stand outside of the sphere of facts, emboldens me to say that the author is himself one of the few men belonging to the old school to be found now-a-days, that his book is written in a language not entirely intelligible and sometimes even repulsive to a student of modern mathematics, and in the characteristic tone peculiar to the men of the old school and altogether at variance with the spirit of modern mathematics. May I hope that the remarks just made shall not have any effect on the great credit to which the work is surely entitled in view of the immense difficulties connected with its compilation.

Mathematics in the old days prior to the middle of the seventeenth century.

I might just as well skip over a period covering more than two thousand years prior to the middle of the seventeenth century, during which time no progress seems to have been made beyond the limit of arithmetic and the rudiments of elementary algebra and geometry. I shall only notice a few things which seem to have had their origin in those ancient days, and survived to this day in some form or other.

The system of numeration seems to have been from the beginning the decimal system with the circulating periods consisting of four digits instead of three. The most of the methods of calculation which are now-a-days included under the general name of elementary arithmetic, seem to have been known from the very early days. No doubt some of these methods had their origin in Japan; at the same time a large number of them seem to have been derived from Chinese sources. For some account of the Chinese arithmetic, with which I shall not have much to do in the sequel, I may refer to an article in one of the early volumes of Crelle's Journal, the article Arithmetic in the Encyclopedia Metropolitana written by Peacock, and some of the histories of mathematics, such as the well known treatise of the president of this section.

Beyond the limits of elementary arithmetic, no essential advance seems to have been made except perhaps the solutions of simple equations and some rough methods of calculating lengths and areas. That the hypothenuse of a right angled triangle whose sides may be represented by 3 and 4 respectively, will be represented by 5, seems to have been known from very remote days. No doubt this was found by experience or by some tentative process. This so-called method of three-four-five is up to this day still used by artisans in testing perpendicularity, in some kinds of rough carpentry work.

Some of the names given to the various methods of calculation are as amusing as they are suggestive of their primitive nature. The summation of an arithmetical progression whose common difference is unity is called the timber-piling-calculation, for which piles of shots may just as well be substituted; again the summation of a geometrical progression whose common ratio is 2, is called the rats-calculation, by which no doubt it is meant to signify that the sum increases with the number of terms at an enormous rate, as rats are proverbially said to increase in the course of time.

In the early days, the actual calculation was done by means of rods, at first made of bamboo and afterwards of wood, but always known by the technical name of bamboo-rods. The numbers from 1 to 9 are designated by means of rods as follows:

\[
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
| & \| & \| | & \| | | & \| & \| | & \| | | | | \\
\end{array}
\]

With these together with a symbol which stood for zero and which
was sometimes replaced by a vacant space, they were able to write down any number, and calculate in exactly the same manner as we now-a-days do with Arabic numerals. It may only be necessary to remark that a rod put diagonally across a number shows that the number is to be subtracted, that two numbers put down one above the other are to be added, that two numbers put down side by side are to be multiplied together, and that two numbers put down on the opposite sides of a certain vertical line are to be divided one by the other.

The use of the bamboo-rods in numerical calculations has since been entirely superseded by the introduction of a kind of abacus called soroban, which took place towards the end of the sixteenth century. The appearance of the soroban is depicted below.\[\star\]

It will not be necessary to describe the soroban, and how calculations are made with it, as no doubt this simple yet handy calculating machine is already well known, and even if this be not the case, how to use this instrument is likely to be at once apparent, it being not much different from other forms of abacus. It is very convenient and indeed, I may say, almost indispensable to those who are accustomed to its use in adding numbers, especially when the numbers to be added are, instead of being written down beforehand, read off in rapid succession. Addition, subtraction and multiplication are done with soroban much in the same way as they are done in the ordinary written arithmetic. Division can also be performed on soroban in the same way as it is done in the ordinary written arithmetic, and indeed it is often done in that way; but here it is customary to make use of a peculiar mnemonic which may be called a division table versus multiplication table. For the account of this mnemonic as well as for a lucid explanation of the performance of calculation on soroban, I can not do better than refer to a paper written by Prof. C. G. Knott, formerly of the Imperial University of Tokio, but now of the University of Edinburgh, which is to be found in the Transactions of the Asiatic Society of Japan published about twelve years ago.\[\star\] Omitted here—ED.

The introduction of soroban was followed by progress, a little beyond the domain of elementary arithmetic. Something of algebra which seems to have gone so far as the rational treatment of negative numbers but could not possibly develop itself to any higher stage on account of its altogether cumbersome notation, something of geometry chiefly concerned with the study of regular polygons, magic squares, etc., seems to have been the subject of favourite study of the mathematicians of this time. How much they really achieved, seems to be a question difficult to answer. It appears, however, highly probable that the rigour of methods and proofs so essential for the systematic development of mathematics was not recognized in all its importance. Most of the results arrived at, seem to have been obtained in a haphazard way by the tentative method of elaborate trials and subsequent verification. In support of this assertion, a case may be cited where the value of \(\pi\) was assumed to be \(\sqrt{10}\), merely on account of the rough agreement of this number with the ratio in question, \(\sqrt{10}\) being 3.162+.

The Pythagorean theorem, likely without any rigorous proof, seems to have been known to the men of this time. In evidence of this, I may quote an instance where the periphery of the 2\textsuperscript{nd} sided regular polygon is calculated to a large number of decimal places, giving for the ratio of the periphery to the radius of the circumscribed circle the number

\[3.141526648779698836248\]

A book which appeared about this time contains a magic square containing the numbers from 1 to 400. It seems that this was obtained by the elaborate wearisome method of trials.

I have now passed over the period of years, which, as before said, I might have just as well skipped, during which it is difficult to discriminate things really indigenous from those derived from Chinese sources, and which after all seem to possess nothing but the interest of curiosity.

The mathematics of the old Japanese school properly so-called.

I now pass on to the most important part of the present note,
NOTE ON THE MATHEMATICS OF THE OLD JAPANESE SCHOOL

namely the mathematics of the old Japanese school properly so-called, which originated in Japan and had its secluded development under no external influence. This school of mathematics was founded and developed to a considerable extent by a mathematician of the name Seki, who was contemporaneous with Newton and Leibniz, in fact, born in the same year as Newton. Seki was surely a man of great mathematical ability and originality. He had a tutor, of whom he might have said what Gauss is said to have spoken of his tutor Pfaff. Seki’s success as the founder of the new school of mathematics is to be traced to the great weight which he, unlike his predecessors, laid on the rigour of the methods and proofs. It is he who gave the first rigorous geometrical proof of the Pythagorean theorem.

The remarks which I have made concerning the various difficulties attending the study of the subject of this note, can not possibly be applied with greater force than to the works of Seki. Most of his discoveries were kept in secret among a few of his disciples in his life time, and were only published from time to time in fragmentary form after his death. I believe that there are still some which have not been published and that some have been totally lost.

Seki seems to have begun his work on algebra by improving the clumsy notations and cumbersome operations with the so-called bamboo-rods which were in vogue up to his time. He was so far successful in as much as every thing and every operation could be written down, so that he could dispense with every thing but pen and paper. Connected herewith, he introduced an improved notation, the advantage of which may be exemplified by the transition from writing the same letter repeatedly in succession to the exponential notation. These happy innovations seem to have lead to numerous important discoveries, including among other things the introduction of imaginary numbers, which resulted in the sudden expansion of the domain of algebra. Seki in his late years was no doubt in possession of most of the algebraic methods which we now-a-days find in a treatise on algebra.

It is difficult to draw any exact line of demarkation between the works of Seki and those of his immediate disciples and successors, as it might be imagined from the circumstances mentioned above, under which Seki’s discoveries were transmitted to posterity.

Among the discoveries made by Seki, or, if not by himself, by his immediate disciples and successors under the influence of their master’s work, may, besides the algebra just mentioned, be enumerated:

1°. Some theorems belonging to the theory of numbers;
2°. Elementary geometry both plane and solid, with special reference to the theory of regular polygons;
3°. Trigonometry, accompanied by the construction of the trigonometric tables;
4°. Elements of analytical geometry;
5°. Calculus of finite differences;
6°. Some idea of limits, infinites and infinitesimals;
7°. The theory, most probably algebraical, of maxima and minima;
8°. Summation of a certain class of series;
9°. The so-called principle of circle, including some methods of rectification, quadrature, and cubature of curves and surfaces.

The so-called principle of circle mentioned last, which, in absence of any suitable name, is literally translated from the original, seems to be the climax of Seki’s discoveries, and is held by men of the old school as comparable with the discovery of infinitesimal calculus by Newton and Leibniz. It appears to consist in an ingenuous application of the idea of limits and the summation of infinite series to problems which now-a-days belong to the geometrical application of integral calculus. My opinion is that the so-called principle of circle is a name given to the aggregate of the various methods of rectification, quadrature and cubature of curves and surfaces without the formal use of differential and integral calculus, very much like the methods which were in vogue prior to the time of Bernoulli and Euler, such as those to be found in the work of Wallis. No doubt this name was given to the method, because it was first found in connection with the rectification of the circle.

Judging from the figures which the mathematics of Seki’s school furnished for the ratio of the radius of the circumscribed circle to the
side of a regular polygon, it seems probable that Seki had found some method of solving binomial equations. The following are some of the figures given:

Number of the sides of a regular polygon. The corresponding ratio in question.

<table>
<thead>
<tr>
<th>Sides</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1.152382435</td>
</tr>
<tr>
<td>11</td>
<td>1.71472766</td>
</tr>
<tr>
<td>13</td>
<td>2.0892490734</td>
</tr>
<tr>
<td>17</td>
<td>2.721095575</td>
</tr>
<tr>
<td>19</td>
<td>3.03776691</td>
</tr>
</tbody>
</table>

(a) Greater than $\frac{1}{2}$ in the last decimal place.
(b) Less than $\frac{1}{2}$ in the last decimal place.
(c) Considerably greater.
(d) Slightly greater.

In this connection, it may be remarked that neither Seki nor his followers were in possession of logarithmic tables.

Following the usage prior to his time, Seki calculated the length of the periphery of the $2^n$ sided regular polygon in terms of the radius of the circumscribed circle as unit, and found $3.1415926538899927759\ldots$

It is told that the comparison of this number with those already found for the $2^5$ and the $2^9$ sided regular polygons lead Seki to the adoption of $3.14159265359$ (slightly less) for the value of $\pi$. It is to be observed that this number is correct to the last decimal place. There are, however, circumstances which suggest that this was merely a stratagem by which the true path of arriving at the above correct value of $\pi$ was kept in secret. Most probably he was in possession of an expression of $\pi$ in the form of an infinite series, derived from the periphery of a regular polygon by making the number of sides infinitely great, by which the value of $\pi$ can be calculated to any desired degree of accuracy. In support of this, it may be remarked that one of the disciples of Seki calculated the value of $\pi$ to 49 decimal places, which value is found to be correct to the last figure.

Seki seems to have known something of continued fractions, as we find among his posthumous work such a method as the reduction of quadratic surds to continued fractions; but it is not at all likely that he was in possession of the expression of $\pi$ in the form of a continued fraction. This may be inferred from the tentative method, by which he arrives at the approximate value of $\pi$ in the form of a fraction. Beginning with 1 over 1, we have to add either 3 to the numerator and 1 to the denominator, or 4 to the numerator and 1 to the denominator, according as the value of the resulting fraction is greater or less than the true value of $\pi$. Continuing this process 113 times, in the course of which we pass through fractions such as $\frac{22}{7}$, we arrive at the well known fraction $\frac{355}{113}$. The expression of $\pi$ in the form of a continued fraction appears, however, to have been found by one of the men belonging to Seki's school sometime after his death, who gives for an approximate value of $\pi$ the fraction

Numerator $428,224,593,349,304$,
Denominator $136,308,121,570,117$.

This fraction gives the value of $\pi$ correct to the 29th decimal place.

I have spoken somewhat at length of the work of Seki on the rectification of the circle. It is, however, by no means meant to indicate that his mathematical activity was confined to this special subject.

I now leave Seki and his immediate disciples, and pass on to speak of some of the men whose works had great influence on the further development of the mathematics of the old school. Foremost among them, stands Yasushima, whose work was mostly done in the latter half of the 18th century. To him is to be attributed a complete theory of integration based upon the summation of infinite series. Yasushima seems to have begun his work in this direction by taking up again the favourite subject of his school, namely the rectification and quadrature of the circle. Unlike his predecessors, who occupied themselves with finding the entire periphery and the entire area of a circle, he showed how to find first the area of any sector and then the length of any arc of a circle. In the following, I shall reproduce his
method with slight modifications conducive to the better understanding.

Let OA and OB be two radii of a circle at right angles to each other. Draw an ordinate CD parallel to OB. It is required in the first place to find the area OBCD.

Let the radius of the circle be denoted by $r$ and the length of OD by $a$. Divide OD into $n$ equal parts and through each point of division draw an ordinate parallel to OB. Let these ordinates be numbered in succession from left to right, so that the last, i.e., CD, becomes the $n^{th}$. Then the length of the $m^{th}$ ordinate will be given by

$$\sqrt{r^2 - \left(\frac{ma}{n}\right)^2}.$$ 

To free this expression of its radical, it is expanded by means of the binomial theorem, which, by the way, seems to have been known since the time of Seki. The result, as may easily be verified, is

$$n\left[1 - \frac{1}{2} m^2 \left(\frac{a}{r}\right)^2 - \frac{1}{8} \frac{m^4}{n^2} \left(\frac{a}{r}\right)^4 - \frac{1}{16} \frac{m^6}{n^4} \left(\frac{a}{r}\right)^6 - \frac{5}{128} \frac{m^8}{n^6} \left(\frac{a}{r}\right)^8 - \frac{7}{256} \frac{m^{10}}{n^{10}} \left(\frac{a}{r}\right)^{10} - \frac{21}{1024} \frac{m^{12}}{n^{12}} \left(\frac{a}{r}\right)^{12} - \cdots \right].$$

Following the usage of his time, Yasushima does not give the general term of the series within the brackets, which may readily be found to be

$$\frac{1.3.5 \cdots (2p-3)}{2^p \cdot p!} \frac{m^{2p}}{n^{2p}} \left(\frac{a}{r}\right)^{2p}.$$ 

Multiplying the above expression by $\frac{a}{n}$ and summing with respect to $m$ from 1 to $n$, we obtain

$$ar\left[1 - \frac{1}{2} \frac{\Sigma m^2}{n^2} \left(\frac{a}{r}\right)^2 - \frac{1}{8} \frac{\Sigma m^4}{n^4} \left(\frac{a}{r}\right)^4 - \cdots \right],$$

where $\Sigma$ denotes summation with respect to $m$ from 1 to $n$. It remains only to make $n$ infinite. The works which are said to contain the account of evaluating $\lim_{n \to \infty} \left(\frac{1}{n^{a+1}}\right)$ were not accessible to me, but it seems to me exceedingly probable that this limiting value was found in the following manner (it is needless to say that this limiting value is equal to $\int_0^1 x^a \, dx$). By the binomial theorem

$$(m+1)^{a+1} - m^{a+1} = (\lambda+1)m^a + \frac{\lambda+1}{12} \lambda m^{a-1} + \cdots$$

Herein put $m=1, 2, 3, \ldots, n$ in succession, and add the resulting equations side by side. We get

$$(n+1)^{a+1} - 1 = (\lambda+1) \sum_{a=1}^{n} m^a + \frac{\lambda+1}{12} \sum_{a=1}^{n} m^{a-1} + \cdots$$

Divide by $n^{a+1}$ and then making $n$ infinite, we obtain

$$\frac{\sum_{a=1}^{n} m^a}{n^{a+1}} \to \frac{1}{\lambda+1}.$$ 

Thus we obtain finally for the required area

$$ar\left[1 - \frac{1}{6} \left(\frac{a}{r}\right)^2 - \frac{1}{40} \left(\frac{a}{r}\right)^4 - \frac{1}{112} \left(\frac{a}{r}\right)^6 - \frac{5}{1152} \left(\frac{a}{r}\right)^8 - \frac{7}{2816} \left(\frac{a}{r}\right)^{10} - \frac{21}{13312} \left(\frac{a}{r}\right)^{12} - \cdots \right],$$

the general term of the expansion within the brackets being

$$\frac{1.3.5 \cdots (2p-3)}{2^p \cdot p! \cdot (2p+1)} \left(\frac{a}{r}\right)^{2p}.$$ 

Again by subtracting from the area of OBCD just found the area of the triangle OCD, i.e.

$$\frac{a\sqrt{r^2-a^2}}{2} = ar\left[1 - \frac{1}{2} \left(\frac{a}{r}\right)^2 - \frac{1}{8} \left(\frac{a}{r}\right)^4 - \frac{1}{16} \left(\frac{a}{r}\right)^6 - \cdots \right],$$

and dividing the result by $r$, we obtain the following expression for the length of the circular arc BC.
NOTE ON THE MATHEMATICS OF THE OLD JAPANESE SCHOOL.

\[
\frac{a}{2} \left[ 1 + \frac{1}{6} \left(\frac{a}{r}\right)^2 + \frac{3}{40} \left(\frac{a}{r}\right)^4 + \frac{5}{112} \left(\frac{a}{r}\right)^6 + \frac{35}{1152} \left(\frac{a}{r}\right)^8 + \frac{63}{2816} \left(\frac{a}{r}\right)^{10} + \frac{231}{13312} \left(\frac{a}{r}\right)^{12} + \cdots \right],
\]

the general term of the series within the brackets being

\[
\frac{1.3.5\cdots(2p-3). (2p-1)}{2^p \cdot p! \cdot (2p+1)} \left(\frac{a}{r}\right)^{2n}
\]

In passing by, it may be observed that the above gives a direct method of expanding \(\text{arcsin} \frac{a}{r}\).

Yasushima gives a peculiar form to the above series; namely, putting this series equal to

\[
\frac{a}{2} (u_0 + u_1 + u_2 + u_3 + u_4 + \cdots)
\]

and \(\left(\frac{a}{r}\right)^2 = A\), he writes

\[
\frac{a}{2} \left( 1 + \frac{1^2 \cdot u_0 A}{2 \cdot 3} + \frac{3^2 \cdot u_2 A}{4 \cdot 5} + \frac{5^2 \cdot u_4 A}{6 \cdot 7} + \frac{7^2 \cdot u_6 A}{8 \cdot 9} + \frac{9^2 \cdot u_8 A}{10 \cdot 11} + \frac{11^2 \cdot u_{10} A}{12 \cdot 13} + \cdots \right).
\]

Proceeding further on, Yasushima makes a fatal mistake by saying that we may obtain the expression for \(\frac{\pi}{4}\) by putting in the above series \(a = r\) and \(r = 1\), because the resulting series diverges. However this is very instructive, as it shows that even such a prominent man of the old school as Yasushima seems to have had no idea of the convergency of infinite series. Nevertheless a correct expression for \(\frac{\pi}{4}\) might have been obtained by dividing the expression of the area of a quadrant by the radius and then putting \(r\) equal to unity. By so doing, he might have obtained

\[
\frac{\pi}{4} = 1 - \frac{1}{6} - \frac{1}{16} - \frac{1}{112} - \frac{5}{2816} - \frac{21}{13312} - \cdots - \frac{1 \cdot 3 \cdot 5 \cdots (2p-3)}{2^p \cdot p! \cdot (2p+1)}
\]

Yasushima also discovered a method of double integration very much on the same line as the method of simple integration exemplified above. All these methods which were still included under the all comprehensive name of the principle of circle, were successfully applied to such problems as the finding of the length of an elliptic arc, the common volume of two intersecting cylinders, and the like.

Yasushima's discoveries which seem to have been of the most varied nature, include among other things a complete theory of spherical trigonometry. It may also be added that he or one of his immediate disciples found a method for the numerical solution of equations.

I now pass on to Wada, who is held as another giant by men of the old school and who published most of his discoveries early in the beginning of the nineteenth century.

Wada's first labour seems to have been directed toward improving the various methods found before his time. To give only an example of his work in this direction, he gives the following improvement on the method of finding the length of a circular arc. From the similarity of the triangles OPQ and PSR in the adjoining figure, we have

\[
\text{OP} : \text{PQ} = \text{PS} : \text{RS};
\]

whence, with the notations already used in giving Yasushima's method, we get

\[
\text{arc PS} = \frac{a}{\sqrt{r^2 - \left(\frac{ma}{n}\right)^2}}.
\]

Further, proceeding in exactly the same way as in the above, we obtain the same result as that found by Yasushima. It is interesting to observe that the method, after passing through the various stages of development, has developed itself to something which is not essentially different from the modern method.

In trying to give some account of the works of some of the most prominent of the men of the old school, I might have given examples other than that of the rectification of a circular arc; but I have purposely confined myself to this particular example, in order to give possibly some idea of the various stages through which the development of this peculiar mathematics seems to have passed.

The most important of Wada's contributions to the mathematics of
the old school is said to consist in publishing various tables which give numerical values of the coefficients of a large number of infinite series connected with rectification, quadrature and cubature. Wada seems also to have been the first mathematician who applied the principle of inequality to the evaluation of limiting values. He tried also to express the length of the various right lines connected with a circular arc in terms of the circular arc, and was successful to a certain extent. Perhaps he was in possession of inverse trigonometric functions in some form or other.

Wada and some of his contemporaries occupied themselves with the study of such curves as the cycloid, the catenary and the like, and it seems as if most of the well known properties of these curves were known to them. The other favourite subjects of about this time seem to have been the calculation of the centers of gravity of some simple figures, magic squares and geometrical contact problems more or less complicated.

About the middle of the nineteenth century, logarithmic tables seem to have been introduced from Dutch sources. We are told that some of the men of the old school were hard at work in calculating the common logarithms of numbers, no doubt by some primitive method, as they were not in possession of the logarithmic series.

About this time, some Dutch mathematical books came into the possession of the mathematicians of the old school. Although they must have had great difficulties in reading Dutch, it is conceivable that these books had some influence on the works of these men. There still remains one thing which I should like to mention before quitting the subject of this note. I was told that the theory of determinants and its application to the solution of the system of linear equations were not entirely unknown to the men of the old school.

Before concluding this brief discourse, may I be permitted to repeat once more what I have said in the beginning. I have been speaking of things which are now entirely obsolete and which can have at most historical interest, leaving however the chance that some really valuable things might still be found in those regions of this mathem-
ON THE METHOD OF THE OLD JAPANESE SCHOOL FOR FINDING THE AREA OF A CIRCLE.

(ABSTRACT OF THE PAPER READ SEPT. 21ST., 1895.)

BY

Prof. D. Kikuchi. (Imperial Univ., Tókyó.)

The diameter $d$ is divided into any number of parts $n$. Put $\frac{d}{n} = a$.

\[ b_1 = d - \frac{1^2 \cdot d}{2 \cdot n^2} - \frac{1^4 \cdot d}{8 \cdot n^4} - \frac{1^6 \cdot d}{16 \cdot n^6} - \frac{1^8 \cdot 5 \cdot d}{128 \cdot n^8} \]

\[ b_2 = d - \frac{2^2 \cdot d}{2 \cdot n^2} - \frac{2^4 \cdot d}{8 \cdot n^4} - \frac{2^6 \cdot d}{16 \cdot n^6} - \frac{2^8 \cdot 5 \cdot d}{128 \cdot n^8} \]

\[ b_3 = d - \frac{3^2 \cdot d}{2 \cdot n^2} - \frac{3^4 \cdot d}{8 \cdot n^4} - \frac{3^6 \cdot d}{16 \cdot n^6} - \frac{3^8 \cdot 5 \cdot d}{128 \cdot n^8} \]

\[ \therefore \frac{A_1}{d^2} + \frac{A_2}{d^2} + \cdots = \frac{1}{n} \sum \frac{\sigma^r \cdot d}{2 \cdot n^2} - \frac{\sigma^r \cdot d}{8 \cdot n^4} - \frac{\sigma^r \cdot d}{16 \cdot n^6} - \frac{\sigma^r \cdot 5 \cdot d}{128 \cdot n^8} \]

Putting $a \times b_1 = A_1$, $a \times b_2 = A_2$, &c., it is evident that the sum of $A_1$, $A_2$, $A_3$, &c., is ultimately, when $n$ is made infinitely large, equal to the area of the circle. Now

\[ \frac{A_1}{d^2} = \frac{1}{n} \frac{1}{2} \frac{1^2}{n^2} - \frac{1^4}{n^4} - \frac{1^6}{n^6} - \frac{1^8}{n^8} \]

\[ \frac{A_2}{d^2} = \frac{1}{n} \frac{2^2}{n^2} - \frac{2^4}{n^4} - \frac{2^6}{n^6} - \frac{2^8}{n^8} \]

\[ \frac{A_3}{d^2} = \frac{1}{n} \frac{3^2}{n^2} - \frac{3^4}{n^4} - \frac{3^6}{n^6} - \frac{3^8}{n^8} \]

\[ \therefore \frac{1}{n} \sum \frac{\sigma^r \cdot d}{2 \cdot n^2} - \frac{\sigma^r \cdot d}{8 \cdot n^4} - \frac{\sigma^r \cdot d}{16 \cdot n^6} - \frac{\sigma^r \cdot 5 \cdot d}{128 \cdot n^8} \]

\[ = 1 - \frac{1}{2.3} - \frac{1}{4} - \frac{1}{12} + \frac{1}{24} \frac{1}{240} \]

\[ = \frac{1}{5.8} \frac{2}{16} - \frac{5}{240} \frac{7.16}{32} \frac{5}{192} \frac{384}{96} \frac{5}{768} \]

\[ - \frac{5}{9.128} - \frac{5}{256} - \frac{5}{192} + \frac{5}{384} \frac{5}{576} \frac{1}{768} \]

Hence, ultimately,

\[ \text{Area of the circle} = 1 - \frac{1}{2.3} - \frac{1}{5.8} - \frac{1}{7.16} - \frac{5}{9.128} \]

The first term on the right hand (1) is styled the original number, the second term the first difference, the 3rd term the second difference, and so on.

First difference is obtained from the original number by multiplying by $\frac{1}{2.3}$; the second difference from the first by multiplying by $\frac{3}{4.5}$ or $\frac{1.3}{4.5}$; the third from the second, by multiplying by $\frac{5}{2.7}$ or $\frac{3.5}{6.7}$; the
fourth from the third by multiplying by \( \frac{5.7}{8.9} \), and so on. These factors are then

\[
\frac{1}{2.3}, \quad \frac{1.3}{4.5}, \quad \frac{3.5}{6.7}, \quad \frac{5.7}{8.9}, \quad \ldots
\]

and the law of the series is evident.

Hence

the original number \(... \ldots \ldots \ldots 1\)

1st difference \(... \ldots \ldots \ldots \frac{1}{2.3} \) (orig. number)

2nd difference \(... \ldots \ldots \ldots \frac{1.3}{4.5} \) (1st diff.)

3rd difference \(... \ldots \ldots \ldots \frac{3.5}{6.7} \) (2nd diff.)

4th difference \(... \ldots \ldots \ldots \frac{5.7}{8.9} \) (3rd diff.)

\[
= 0.7853981633974483096156608458 (+)^* \]

\[
= \text{the area of the circle} \left( = \frac{\pi}{4} \right).
\]

* This is as far as is given in Hasegawa's "Kyusaki Tsukō."
Ê. KIKUCHI, VARIOUS SERIES FOR π

Denote \( \frac{d}{d'} \) by \( λ \).

Then we have
\[
D_{m} : D_{n} = OD_{m} : OD_{n} ;
\]
\[
\therefore \quad D_{m} = \frac{\epsilon}{2n} \cdot \frac{d}{2} \left( \frac{d^{3}}{4} - \frac{m^{3}}{4n^{2}} \right) = \frac{\epsilon}{2n} \left( 1 + \frac{m^{2}}{n^{2}} \lambda^{2} + \frac{m}{2} \lambda^{3} + \cdots \cdots \right)
\]

If we take twice the sum of \( D_{m} \) for the values of \( m \) from 1 to \( n \) and then make \( n \) indefinitely large, we get the arc required. Hence,
\[
\text{the arc} = \epsilon \left( 1 + \frac{1}{2.3} \lambda^{3} + \frac{3}{8.5} \lambda^{4} + \frac{15}{48.7} \lambda^{5} + \frac{105}{384.9} \lambda^{6} + \cdots \cdots \right)
\]
\[
= \epsilon \left( \text{the original number} \right) + \frac{1}{2.3} \lambda^{3} \quad \text{(orig. number)} + \frac{3}{8.5} \lambda^{4} \quad \text{(1st diff.)} + \frac{15}{48.7} \lambda^{5} \quad \text{(2nd diff.)} + \frac{105}{384.9} \lambda^{6} \quad \text{(3rd diff.)} + \frac{15}{89} \lambda^{7} \quad \text{(4th diff.)} + \cdots \cdots \right)
\]

(A)

The noteworthy points in the above are
(a) Taking the length of the tangent \( D_{m} \) as ultimately equal to the arc;
(b) In the Enri Shinkō, this length is called \( kai kaku \) (掛看) i.e. difference of the arc, which is equivalent to \( dx \);
(c) Using the general \( (n\text{th}) \) term, in the original 槁, meaning "a certain," as 槉看 a certain chord, for \( n\text{th} \) chord;
(d) Denoting \( \frac{d}{d'} \) by \( λ \) or rather in the original \( \frac{d}{d'} \) by 看, meaning constant or modulus. \( \frac{d}{d'} \) is the sine of half the arc. (In the old Japanese Mathematics we do not find the use of circular functions.)

The above series may be written
\[
\epsilon \left( 1 + \frac{1.1}{2.3} \lambda^{3} + \frac{1.1.3}{2.4.5} \lambda^{4} + \frac{1.1.3.5}{2.4.6.7} \lambda^{5} + \cdots \cdots \right)
\]
\[
= \epsilon F \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \lambda \right) \quad \text{according to Gauss' notation for hypergeometrical series.}^{9}
\]

If the arc be \( d' \), we have, from this,
\[
\theta = \sin \phi F \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \sin \theta \right)
\]
(B)
a well-known series, due to Newton.

From this we deduce the value of \( \pi \) by putting \( \epsilon = d' \), viz:
\[
\pi = 2F \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, 1 \right)
\]
II.

The series given in the last paper for \( \frac{\pi}{4} \) was
\[
1 - \frac{1}{2.3} - \frac{1.1}{2.4.5} - \frac{1.1.3}{2.4.6.7} - \cdots \cdots
\]
\[
= F \left( -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, 1 \right)
\]
I.

These two series may be derived easily one from the other.

The area of the segment of the circle, given the diameter and the chord.

\[ A_{m} \] is the area of the \( m\text{th} \) rectangle, i.e., contained by the \( m\text{th} \) chord and the difference of the \( m\text{th} \) and \( (m-1)\text{th} \) ordinates

\[
= 2Od_{m} \cdot OD_{m} = 2Od'_{m} \cdot OD'_{m}
\]
\[
= 2.\frac{m^{3}}{2n^{2}} \sqrt{\frac{d}{2} \left( 1 - \frac{m^{2}}{n^{2}} \right)}
\]

I. Gauss, On the Infinitesimal Series \( 1 + \frac{a}{b} \frac{x}{y} + \frac{a(a+1)b(b+1)}{1.2.3(y+1)} \cdots \cdots \cdots \cdots ; \)

This Kijii, Vol. IV, or Gaussamete Werke, Vol. III.

II. This series is given in the Enri Shinkō, directly from the consideration of the semi-circumference by the same method.

III. The ordinates are called \( y \) (lengths), and, this difference is called 槉看, a certain-length-difference, equivalent to \( dy \).
D. KIKUCHI, VARIOUS SERIES FOR $\pi$

$$= \frac{\pi^2}{2d^2} \left( 1 + \frac{1}{2} \frac{\mu}{d} + \frac{3}{8} \frac{\mu^3}{d^3} + \frac{15}{48} \frac{\mu^5}{d^5} + \cdots \right)$$

We have to take the sum of $A_n$ from $m=1$ to $m=n$ and then make $n$ indefinitely large. We thus get the area of the segment

$$= \frac{\pi^2}{2d^2} \left( \frac{1}{3} + \frac{1}{2} \frac{\mu}{d} + \frac{3}{8} \frac{\mu^3}{d^3} + \frac{15}{48} \frac{\mu^5}{d^5} + \frac{105}{384} \frac{\mu^7}{d^7} + \cdots \right)$$

$$= \frac{\pi^2}{2d^2} \text{the original number} + \frac{1}{2} \frac{\mu}{d} \text{ (orig. number)} + \frac{3}{8} \frac{\mu^3}{d^3} \text{ (1st diff.)} + \frac{15}{48} \frac{\mu^5}{d^5} \text{ (2nd diff.)} + \cdots$$

$$= \frac{\pi^2}{2d^2} \left( \frac{1}{2} + \frac{1}{2} \frac{\mu}{d} + \frac{3}{8} \frac{\mu^3}{d^3} + \frac{15}{48} \frac{\mu^5}{d^5} + \cdots \right)$$

$$= \frac{\pi^2}{2d^2} \left( \frac{1}{2} + \frac{1}{2} \frac{\mu}{d} \text{ (orig. number)} \right)$$

$$= 2\sqrt{s d} \left( \frac{1}{2} + \frac{1}{2} \frac{\mu}{d} \right) \left( \frac{1}{2} + \frac{1}{2} \frac{\mu}{d} \text{ (orig. number)} \right)$$

Putting $s=\frac{1}{2} d$, we get the value of $\pi$, viz:

$$\pi = 2\sqrt{s d} \left( \frac{1}{2} + \frac{1}{2} \frac{\mu}{d} \right)$$

which may be obtained at once by putting $\phi = \frac{\pi}{4}$ in the expansion of $\phi$ in terms of $\sin \phi$.

The area of the segment

$$= 2 \left( \text{the segment corresponding to the chord } \sqrt{s d} \right)$$

$$+ \left( \text{the triangle, altitude } s, \text{ base } 2\sqrt{s d - s^2} \right)$$

$$= s\sqrt{s d} \left( \frac{1}{3} + \frac{1}{2.5} \mu + \frac{3}{8.5} \mu^3 + \frac{15}{48.9} \mu^5 + \frac{105}{384.11} \mu^7 + \cdots \right)$$

$$= s\sqrt{s d} \left( \frac{1}{3} - \frac{1}{2.4} \mu + \frac{1.3}{2.4.6} \mu^3 - \frac{1.3.5}{2.4.6.8} \mu^5 + \cdots \right)$$

$$= s\sqrt{s d} \left( \frac{1}{3} - \frac{1}{2.5} \mu + \frac{3}{8.7} \mu^3 + \frac{1.3.5}{2.4.6.8.11} \mu^7 + \cdots \right)$$

$$= 4s\sqrt{s d} \left( \frac{1}{3} - \frac{1}{2.5} \mu + \frac{3}{8.7} \mu^3 + \frac{1.3.5}{2.4.6.8.11} \mu^7 + \cdots \right)$$

$$= 4s\sqrt{s d} \left( \frac{1}{3} + \frac{1}{2.5} \mu + \frac{3}{8.7} \mu^3 + \frac{1.3.5}{2.4.6.8.11} \mu^7 + \cdots \right)$$

$$= \frac{4}{3} s\sqrt{s d} \text{the original number}$$

$$- \frac{3}{2.5} \mu \text{ (orig. number)}$$

$$- \frac{1.5}{4.7} \mu \text{ (1st diff.)}$$

$$- \frac{3.7}{6.9} \mu \text{ (2nd diff.)}$$

$$- \frac{13.5}{2.4.6.8.11} \mu^7 + \cdots$$

The arc and segment of a circle, given the diameter and the sagitta.

Let the sagitta $= s$.

Then the chord of half the arc $= \sqrt{s d}$. Hence, to find the arc, put $\sqrt{s d}$ for $c$ in the series (A) and double the result.

Also, put $\frac{\sqrt{s d}}{d} = \sqrt{\frac{s}{d}} = \sqrt{\frac{\mu}{d}}$ for $\frac{c}{d}$ or $\lambda$.

The arc

---

\(^{1)}\) This can be obtained easily by substituting for $\phi$ and $\cos \phi$ in terms of $\sin \phi$ or $\frac{c}{d}$ in the expression $\frac{c^2}{4}$ ($\phi - \sin \phi \cos \phi$) for the area.
D. KIRUCHI, VARIOUS SERIES FOR \( \pi \)

\[-\frac{5.9}{8.11} \mu \text{ (3rd diff.)} \quad \text{the 4th difference.} \]

\[= \frac{4}{3} \sqrt{5} F \left( \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{6}{2} \right) \quad \text{(E)} \]

Whence putting \( s = \frac{d}{2} \) we get the value of \( \pi \), viz:

\[\pi = 8\sqrt{\left( \frac{1}{3} - \frac{1}{2}, \frac{2.5}{2^2}, \frac{2.4.7}{3^3}, \frac{1.3.5}{2.4.6}, \frac{1.3.5}{2.4.6.8.11} \right)} \]

\[= \frac{8\sqrt{2}}{3} F \left( \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{1}{2} \right) \quad \text{V.} \]

The expressions for the circumference and the area of the circle are also obtained in the Enri Shinkō from that for the arc (A), by putting \( \epsilon = \frac{1}{2} \), and multiplying by 6 for the arc and by \( \frac{3}{2} \) for the area. Thus the circumference

\[= 3d \left( 1 + \frac{1}{2.3} \frac{1}{4} + \frac{1.3}{2.4.5} \frac{1}{4^2} + \frac{1.3.5}{2.4.6} \frac{1}{4^3} + \right) \]

\[= 3d \quad \text{the original number} \]

\[+ \frac{1^2}{1 \cdot 3.8} \quad \text{(orig. number)} \quad \text{the 1st difference} \]

\[+ \frac{3!}{2.5.8} \quad \text{(1st diff.)} \quad \text{the 2nd difference} \]

\[+ \frac{5^2}{3.7.8} \quad \text{(2nd diff.)} \quad \text{the 3rd difference} \]

\[+ \frac{7^2}{4.9.8} \quad \text{(3rd diff.)} \quad \text{the 4th difference} \]

\[+ \text{ \ldots ...} \quad \text{(F)} \]

and the area

\[= \frac{3d^2}{4} \left( 1 + \frac{1}{2.3} \frac{1}{4} + \frac{1.3}{2.4.5} \frac{1}{4^2} + \right) \quad \text{(G)} \]

From either (F) or (G), we have

\[\pi = 3 \left( 1 + \frac{1}{2.3} \frac{1}{4} + \frac{1.3}{2.4.5} \frac{1}{4^2} + \right) \]

which will follow at once by putting \( \phi = \frac{\pi}{6} \) in (B).

It is to be remarked that the above series for \( \pi \) are all very slowly convergent, and the calculation of \( \pi \) from any of them is very laborious. In the Hōen Sankyō \(^8\) the value of \( \pi \) is given to 50 figures, all correct, as calculated by the author from the series VI.

\(^8\) '新鏡鏡', by Y. Matsumoto (新永鏡鏡), manuscript, circa 1739.
A SERIES FOR $\pi^2$ OBTAINED BY THE OLD JAPANESE MATHEMATICIANS.

BY

Prof. D. Kikuchi. (Imperial Univ., Tokyö.)

In the preceding papers, I gave various series for $\pi$, obtained by the Japanese Mathematicians; the methods there given are the latest developments attained by them. In this paper, I shall give one of the earliest methods by which not the value of $\pi$ but of $\pi^2$ was found, in the form said to be essentially due to Seki (died 1708) the founder of Japanese Mathematics. I follow Yamaji's "Ken Kon no Maki" and another manuscript book entitled Kolai no Ri (Theory of the length of arcs) which goes over the same ground more in detail and is an amplification of the former.

The method in short consists in this. The diameter of the circle and the sagitta of the arc are given, call them $d$ and $s$. The sagittae of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ... arcs are found as series in ascending powers of $s/d$; then the squares of the chords of these arcs are equal to the products of the diameter and the respective sagitta; and so the successive approximations to the arc are obtained.

1. Given the diameter $d$ and the sagitta $s$ of an arc of a circle, to find the sagitta $s_1$ of $\frac{1}{2}$ arc.

$$4(d-s_1) = s_1 = \text{sq. of the chord of } \frac{1}{2} \text{ arc} = sd$$

$$s_1 = s_1d + s \frac{1}{2} sd = 0$$

whence $s_1$ is found to be

$$\frac{s}{4} + \frac{1}{16} \frac{s^3}{d^3} + \frac{1}{32} \frac{s^5}{d^5} + \frac{5}{256} \frac{s^7}{d^7} + \frac{7}{512} \frac{s^9}{d^9} + \frac{21}{2048} \frac{s^{11}}{d^{11}} + \ldots$$

$$= \frac{s}{4}$$

2. Next, the sagitta $s_2$ of $\frac{1}{4}$ arc is obtained from the equation

$$s_2^2 - s_2d + \frac{s_2}{2}d = 0$$

or

$$s_2^2 - s_2d + \left(\frac{s_2}{16} + \frac{s_2^3}{64} + \frac{s_2^5}{128d} + \frac{s_2^7}{1024d^3} + \ldots\right) = 0$$

by the same method; viz:

$$s_2 = \frac{s}{16}$$

<table>
<thead>
<tr>
<th>Term</th>
<th>Original Number</th>
<th>1st Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+\frac{1}{4} \frac{s}{d}$</td>
<td>(orig. number)</td>
<td>the 1st difference</td>
</tr>
<tr>
<td>$+\frac{1}{2} \frac{s}{d}$</td>
<td>(1st diff.)</td>
<td>2nd</td>
</tr>
<tr>
<td>$+\frac{5}{8} \frac{s}{d}$</td>
<td>(2nd diff.)</td>
<td>3rd</td>
</tr>
<tr>
<td>$+\frac{7}{10} \frac{s}{d}$</td>
<td>(3rd diff.)</td>
<td>4th</td>
</tr>
<tr>
<td>$+\frac{3}{4} \frac{s}{d}$</td>
<td>(4th diff.)</td>
<td>5th</td>
</tr>
<tr>
<td>$+\frac{11}{14} \frac{s}{d}$</td>
<td>(5th diff.)</td>
<td>6th</td>
</tr>
<tr>
<td>$+\frac{13}{16} \frac{s}{d}$</td>
<td>(7th diff.)</td>
<td>7th</td>
</tr>
</tbody>
</table>

---

2) 有容之秘, Volumes of Ken and Kon, manuscript said to have been kept secret except from the initiated few, transcribed to Y. Yamaji (山名義直), circa 1785.
3) The method of solving such equations is due to Seki. I reserve its description for a future paper.
3. In this way similar expressions are obtained for \( s_n \), \( s_{n+1} \), \ldots \ldots ..., the sagittae for \( \frac{1}{3} \theta_n \), \( \frac{1}{3} \theta_{n+1} \), \ldots \ldots ..., arcs. And from these values, the law of the series is found.

Namely, in all of them, original number is the sagitta \( s \) divided by the sq. of the number into which the arc is divided; the successive differences are obtained by multiplying the preceding term by \( \frac{s}{d} \) and a numerical factor. The numerical factors are

<table>
<thead>
<tr>
<th>1st diff.</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>3.5</td>
<td>7.9</td>
</tr>
<tr>
<td>3.4</td>
<td>6.8</td>
<td>12.16</td>
</tr>
<tr>
<td>3.5</td>
<td>7.9</td>
<td>15.17</td>
</tr>
<tr>
<td>5.6</td>
<td>10.12</td>
<td>20.24</td>
</tr>
<tr>
<td>5.7</td>
<td>11.13</td>
<td>23.25</td>
</tr>
<tr>
<td>7.8</td>
<td>14.16</td>
<td>28.32</td>
</tr>
<tr>
<td>7.9</td>
<td>15.17</td>
<td>31.33</td>
</tr>
<tr>
<td>9.10</td>
<td>18.20</td>
<td>36.40</td>
</tr>
<tr>
<td>9.11</td>
<td>19.21</td>
<td>39.41</td>
</tr>
<tr>
<td>11.12</td>
<td>22.24</td>
<td>44.19</td>
</tr>
<tr>
<td>11.13</td>
<td>23.25</td>
<td>47.19</td>
</tr>
<tr>
<td>13.14</td>
<td>26.28</td>
<td>52.56</td>
</tr>
<tr>
<td>13.15</td>
<td>27.29</td>
<td>55.57</td>
</tr>
<tr>
<td>15.16</td>
<td>30.32</td>
<td>60.64</td>
</tr>
</tbody>
</table>

The factor for \( n \)th difference in \( \frac{1}{n} \) arc is seen to be

\[
\frac{(mn-1)(mn+1)}{(mn+n\frac{n}{2})(mn+n)} \quad \text{or} \quad \frac{2(mn^2-1)}{m(2m+3)+1}\]

and hence when \( n \) is made indefinitely large, the factor is \( \frac{2m^2}{2m^3+3m+1} \).

4. The chords corresponding to the sagittae \( s_1 \), \( s_2 \), \ldots \ldots ..., the successive approximations to the arc are \( 2c_1 \), \( 2c_2 \), \( 2c_3 \), \ldots \ldots ..., But for any sagitta \( s \), where \( d = n \), the chord \( c_{n+1} = \sqrt{s_n d} \) and the approximation to the arc is \( 2^{n+1} \sqrt{s_n d} \) or \( 2^n c_n \), and therefore the square of the arc is \( 4^{n+1} s_n d = 4^n c_n d \). Substituting for \( s_n \), we get a series of which the first term is \( 4ds \) and \( n \)th difference is derived from the preceding by multiplying by \( \frac{2(mn-1)(mn+1)}{(2m^2+3m+1)n^2} \). Finally the series for the square of the length of the arc is obtained; the first term is

\[
\frac{2m^2}{2m^3+3m+1} \]

4) This also is due to Seki according to "Kohai no Ri."
AJIMA'S METHOD OF FINDING THE LENGTH OF AN ARC OF A CIRCLE.

BY

Prof. D. Kikuchi.

(Imperial University, Tôkyô).

The expressions for \( b_1, b_2, \ldots \) are found in the same way, as given in pp. 24, 25. Namely,

\[
\begin{align*}
b_1 &= C_1C_1' = d - \frac{1^2a'd^4}{2a'^4} + \frac{1^4a'd^4}{8a'^4} - \frac{3^4a'd^4}{48a'^4} \\
b_2 &= C_2C_2' = d - \frac{2^2a'd^4}{2a'^4} + \frac{2^4a'd^4}{8a'^4} - \frac{3^3a'd^4}{48a'^4} \\
b_3 &= C_3C_3' = d - \frac{3^2a'd^4}{2a'^4} - \frac{3^4a'd^4}{8a'^4} - \frac{3^3a'd^4}{48a'^4}
\end{align*}
\]

\[
\therefore \quad b_1 + b_2 + b_3 + \ldots = nd - \frac{\Sigma t^2a'd^4}{2a'^4} + \frac{\Sigma t^4a'd^4}{8a'^4} - \frac{3\Sigma t^4a'd^4}{48a'^4}
\]

Multiply by \( a \) and put for \( \Sigma t^2 \) etc. their values; remembering that \( na = c \), we get

\[
a(b_1 + b_2 + b_3 + \ldots) = cd - \frac{2 \cdot \frac{a'd^4}{a'^4}}{\frac{d^4}{a'^4} - \frac{3 \cdot \frac{a'd^4}{a'^4}}{\frac{d^4}{a'^4} - \frac{a'd^4}{\frac{d^4}{a'^4}}} \times \frac{1}{12}}
\]

\[
= \frac{6 \cdot \frac{a'd^4}{a'^4} - 5 \cdot \frac{a'd^4}{a'^4} + 10 \cdot \frac{a'd^4}{a'^4} - \frac{a'd^4}{a'^4} \times \frac{1}{240}}{\frac{d^4}{a'^4}}
\]

(In the original paper, terms up to \( \Sigma t^{12} \) are given.)

Ultimately, \( v \) is made indefinitely large and consequently \( a \) indefinitely small. The left-hand side becomes the area \( A \) of the figure \( C_1C_1'D_1'C \) consisting of two equal segments of the circle with a rectangle between. On the right-hand side, all the terms except the first in each line become zero. Hence,

\[
A = cd - \frac{\frac{a'd^4}{a'^4}}{\frac{d^4}{a'^4} - \frac{3 \cdot \frac{a'd^4}{a'^4}}{\frac{d^4}{a'^4} - \frac{a'd^4}{a'^4}} \times \frac{1}{12}}
\]

The area of the rectangle \( CDD'C' = \beta \)

\[
= 4 \text{ area of the triangle } OCD
\]

\[
= cd - \frac{\frac{a'd^4}{a'^4}}{\frac{d^4}{a'^4} - \frac{3 \cdot \frac{a'd^4}{a'^4}}{\frac{d^4}{a'^4} - \frac{a'd^4}{a'^4}}}
\]
AJIMA'S METHOD OF FINDING

[From these, the area of the segment can be obtained at once, being equal to \( \frac{1}{2} (A - B) \); the result will be series (C) of p. 50.]

\[ 2A - B = 4 \text{ of the sector } COL. \]

Hence, the length of the arc CD

\[ \frac{2A - B}{d} = \epsilon + \frac{\epsilon^3}{6d^2} + \frac{\epsilon^4}{40d^4} + \frac{5\epsilon^5}{112d^5} + \frac{35\epsilon^6}{1152d^6} + \cdots. \]

This is the same as the series (A) of p. 48.

\( s \) being the sagitta of the arc, the arc is found in terms of \( s \) and \( d \) as in the series (D) of p. 51.

Again, \( \epsilon^4 = 4(sd^2 - s^2) = 4d(d - s) \),

whence \( \frac{\epsilon^4}{d^2} = \frac{4s}{d} - \frac{4s^2}{d^2} \).

Hence, the arc is

\[ \epsilon + \frac{4s\epsilon}{d} - \frac{4s^2\epsilon}{d^2} \times \frac{1}{6} \]

\[ + \frac{16s^2\epsilon}{d^3} - \frac{32s^3\epsilon}{d^4} + \frac{16s^4\epsilon}{d^5} \times \frac{3}{40} \]

\[ + \frac{64s^3\epsilon}{d^4} - \frac{192s^4\epsilon}{d^5} + \frac{192s^5\epsilon}{d^6} - \frac{64s^6\epsilon}{d^7} \times \frac{5}{112} \]

\[ + \cdots \]

\[ = \epsilon + \frac{2s}{3d} + \frac{8s}{15d^2} + \frac{16s}{35d^3} + \frac{128s}{315d^4} + \frac{156s}{693d^5} + \cdots \]

\[ = \epsilon + \frac{2}{3} \left( \frac{s}{d} \right) \] (the original number)

\[ + \frac{4}{5} \left( \frac{s}{d} \right) \] (the 1st difference)

\[ + \frac{6}{7} \left( \frac{s}{d} \right) \] (the 2nd difference)

\[ + \frac{8}{9} \left( \frac{s}{d} \right) \] (the 3rd difference)

\[ + \frac{10}{11} \left( \frac{s}{d} \right) \] (the 4th difference)

THE LENGTH OF AN ARC OF A CIRCLE.

\[ = \epsilon \left( 1 + \frac{2}{3} \frac{s}{d} + \frac{2.4}{3.5} \frac{s^2}{d^2} + \frac{2.4.6}{3.5.7} \frac{s^3}{d^3} + \frac{2.4.6.8}{3.5.7.9} \frac{s^4}{d^4} + \cdots \right) \]

\[ = \epsilon R \left( 1, 1, \frac{3}{2}, \frac{5}{2} \right). \]

In this, putting \( \epsilon = d, \frac{s}{d} = \frac{1}{2} \), we get

\[ \frac{\pi}{2} = 1 + \frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \frac{1.2.3.4}{3.5.7.9} + \cdots \]

\[ = R \left( 1, 1, \frac{3}{2}, \frac{1}{2} \right) \]

VII. a well known series.

I take this opportunity of expressing my obligations to Mr. T. Endo, for placing at my disposal many of the manuscript books and papers in his possession, and for other assistance which he has kindly given me. I am glad to mention here that his history of Japanese mathematics (in Japanese) is in press and will shortly be published through the liberality of Mr. Hachirōemon Mitsui.
SEKI’S METHOD OF FINDING THE LENGTH
OF AN ARC OF A CIRCLE.

IV

D. Kikuchi,

In Vol. VII of this Kiji, I gave a series for \( \pi \) obtained by the old Japanese mathematicians. I propose now to give an abstract of a manuscript book\(^1\) called Enri Tetsujutsu (図解経術), said to be by Katahiro Takebe (野部賢弘), a disciple of Seki, in which is explained in detail the process by which the series was obtained by Seki.

The following problem is proposed:

At the figure, two-chords are inscribed in a segment, then “four-chords,” “eight-chords,” etc., required the respective sagittae, that is, the original arc is bisected and then bisected again, and so on, required to find the sagittae of the respective arcs.

By the right angled triangles, (長股徑), gen being the hypotenuse, kô the smallest side, kô the third side of a right-angled triangle, small kô (which is

the original sagitta) \( \times \) great gen (the diameter) is equal to the great kô (two-chord) \( \times \) the small gen (two-chord); hence the product of the diameter and the original sagitta is equal to the square of the chord (if half the arc).

Given the diameter [call it \( d \)] and the original sagitta [\( s \)]; required the first sagitta [\( s_1 \)].

The required sagitta is put down as unity, called Ten-gen-no-iichi (天元之一); this was always done by the old mathematicians — i.e. the unknown quantity was so called. Subtract it from the diameter, multiply by \( 4 \); the result is sq. of the chord, and therefore equal to the product of the diameter and the original sagitta. Properly of course it is \( 4 \) times the product of the difference of the diameter and the 1st sagitta into the 1st sagitta, which is equal to the sq. of the chord, but the first sagitta being unity, it is stated as above by the author.

This gives us the quadratic eqn. for \( s_1 \) viz,

\[
4(d-s) s_1 = ds
\]

Before going further I shall make a few remarks on the notation of Japanese algebraists. The Japanese being written vertically downwards, mathematical formulae also were written vertically. Numerical coeffis (not denominators of fractions) were denoted by number of strokes; an oblique stroke was drawn across these strokes to denote minus; letters or rather Chinese ideographs (usually in very abbreviated form) were put by the sides of the strokes, right hand side for multipliers, and left hand side for divisors. There was no sign for equality, one side of an eqn being always 0. Thus the above eqn would be written

\[
\begin{align*}
\text{top:} & \quad \frac{k\bar{h}}{n} + 4d & \text{bottom:} & \quad \frac{\bar{h}}{n} - 4
\end{align*}
\]

---

\(^1\) When I say manuscript book, I do not mean the original manuscript of Takebe; higher parts of Japanese mathematics were kept secret from the general public and only transmitted from master to pupil who equaled manuscript papers of his master and in turn transmitted it to his pupil sometimes with additions of his own, sometimes without any. The copy is the University library is copied from one in possession of Mr. T. Endô; there are some notes purporting to have been added by T. Honda, one of the old masters of Japanese mathematics.
As another example, I write below the expression for the series found
for $s_1$ (see below)

\[
\frac{3}{4} + \frac{5^2}{16d} + \frac{5^3}{32d^3} + \frac{5^4}{256d^4} + \frac{5^5}{512d^5} + \frac{215}{2048d^6} + \frac{33s^7}{4096d^7} + \ldots
\]

I stated that the numerical coefficients (not denominators) were denoted
by strokes; the denominators of fractions were written down in Chinese characters
on the left of the strokes. The way of writing these strokes will be plain
from a glance at the above expression; 1st three terms having only one stroke need nô explanation; the 4th has 5 strokes for 5; the 5th has one vertical stroke above two vertical strokes, the horizontal stroke here counts for 5 so that the coefficient is 7; the 6th has 2 horizontal strokes to the left of 1 vertical stroke, these two horizontal strokes count each for 1 in tens’ place, so that the strokes stand for 21; the strokes for the 7th term mean 33 in the same way; in the 8th, we have 4 vertical strokes to the left standing for 400, next 2 horizontal standing for 20, and 4 vertical below one horizontal stroke on the right standing for 9; in the 9th and 10th terms strokes mean 715 and 2431 respectively; while in the 11th we note that in tens’ place where a horizontal stroke counts for 10, the vertical stroke above the horizontal strokes means 50, making 90 altogether. Thus we have alternately vertical and horizontal strokes to denote units, tens, hundreds, etc; horizontal or

vertical stroke put above these standing for five, fifty, five hundred, etc, $o$ was used for zero. This cumbersome notation derived from the Chinese Mathematics $^3$ was however gradually abandoned, both numerators and denominators being written down in characters. $s$ is abbreviation for 徑 (diameter) In the 2nd term we have 矢巾 (or $s$ once) for 矢之乘乘 which means sagitta once multiplied by itself, i.e. $s^2$; in the 3rd 矢再 (or $s$ twice) for 矢之再乘乘 or $s$ multiplied twice by itself i.e. $s^3$; in the 4th, 5th, etc. we have 矢三 (3·), 矢四 (4·), etc., meaning $s^4$, $s^5$, etc. So for the powers of $\frac{3}{4}$ in the denominators.

To come back to the equation; the solution is given as follows (I have tried to follow the original as closely as translation will allow):

1st. Divide the original [term of] jitsu$^9$ (second column) by the first [term of] $h_0$ (third column), and we get the original [or first term of] shō
(the 1st column).

\[
\text{[ds + 4d = $\frac{3}{4}$]}
\]

2nd. Multiply the original shō into ren (4th column) and write the product in the $h_0$ column, making second [term of] $h_0 \left[ \frac{3}{4} - 4 = -s \right]$.

3rd. Multiply the original shō into the first $h_0$ and cancel against the original jitsu.

4th. Multiply the original shō into the second $h_0$; write the product in the jitsu column, making 1st difference of jitsu$^9$.

5th. Multiply the original shō into ren; and add to the second $h_0$.

Thus:

<table>
<thead>
<tr>
<th>$\text{Sō}$ (result)</th>
<th>$\text{Jitsu}$ (terms not containing the unknown)</th>
<th>$\text{Hō}$ (coeffs. of the 1st power of the unknown)</th>
<th>$\text{Ren}$ (coeffs. of the sq. of the unknown)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>$-ds + ds$</td>
<td>$+4d$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>$-s^3$</td>
<td>$-s \left( = -2s \right)$</td>
<td></td>
</tr>
</tbody>
</table>


$^4$ The original jitsu is put minus, but the quotient is put positive.

$^4$ For the "original" and "difference," see this kij, Vol VII p. 25.
THE LENGTH OF AN ARC OF A CIRCLE.

The original jitsu is now cancelled; we then seek the first difference (term) of the result, as follows:

1st. Divide the first difference (term) of jitsu by the first term of hō,
\[
\frac{s^2}{4} + 4d = \frac{s^2}{16d}
\]
and we get the first difference of shō.

2nd. Multiply the first diff. of shō into ren and we get the 3rd term of hō,
\[
\left[ \frac{s^2}{16d} \times -4 = \frac{s^3}{4d} \right]
\]

3rd. Multiply the first diff. of shō into the first term of hō and cancel against the first difference of jitsu.

4th. Multiply the same into the 2nd term [now—2s] of hō and we get the 2nd diff. of jitsu.

5th. Multiply the same into the 3rd term of hō, and we get the 3rd diff. of jitsu.
\[
\left[ \frac{s^2}{16d} \times -s^3 = \frac{s^4}{64d} \right]
\]

6th. Multiply the same into ren and add to the 3rd term of hō,
Thus:

<table>
<thead>
<tr>
<th>Shō</th>
<th>Jitsu</th>
<th>Hō</th>
<th>Ren</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{s}{4} )</td>
<td>0</td>
<td>+4d</td>
<td>-4</td>
</tr>
<tr>
<td>+ ( \frac{s^2}{16d} )</td>
<td>(-\frac{s^2}{4} + \frac{s^2}{4} )</td>
<td>-2s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-\frac{s^3}{8d} )</td>
<td>(-\frac{s^4}{64d} )</td>
<td>(-\frac{s^4}{2d} )</td>
</tr>
</tbody>
</table>

The author proceeds in this way, repeating the same, step-by-step up to the 9th difference of shō, or the 10th term of the series for the first sagitta; he says, the 10th and further differences might be calculated in the same way; having done so, the sum of the original term and the successive differences is the 1st sagitta, and here he gives the series given above as an example of the notation. The 10th difference, etc., however can, he says, be found more easily by the method given below.

---

SEKI’S METHOD OF FINDING

To find the law of the differences of the first sagitta.

In order to get the result of solution of the quadratic equation, it is not necessary to proceed to more than 5 or 6 terms for there is a certain unchanging law running right through, so that to try to find many terms [as above] is to labour without gain. Let us find the law in the case of the first sagitta, from the first 6 or 7 terms. Divide the 1st difference by the original term, 2nd difference by the 1st difference, 3rd difference by the 2nd, etc.

We have

<table>
<thead>
<tr>
<th>Original Term</th>
<th>( \frac{s}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quotient 1</td>
<td>( \frac{s}{4d} )</td>
</tr>
<tr>
<td>Quotient 2</td>
<td>( \frac{s}{2d} )</td>
</tr>
<tr>
<td>Quotient 3</td>
<td>( \frac{s}{8d} )</td>
</tr>
<tr>
<td>Quotient 4</td>
<td>( \frac{7s}{10d} )</td>
</tr>
<tr>
<td>Quotient 5</td>
<td>( \frac{5s}{4d} )</td>
</tr>
<tr>
<td>Quotient 6</td>
<td>( \frac{11s}{14d} )</td>
</tr>
</tbody>
</table>

Looking at these quotients, we see that the denominators are even numbers and numerators odd, but there seems to be no law. If however we multiply the numerators and denominators of the first quotient by 1, of the second and the fifth by 3, we have a regular order; for the first quotient has 1 in the numerator and 4 in the denominator, the second has 3 in the numerator and 6 in the denominator, the fifth 9 and 12 respectively; thus we have in the numerators a series of odd numbers 1, 3, 5, 7, 9, 11, and in the denominators a series of even numbers 4, 6, 8, 10, 12, 14. Besides these, every quotient has the factor s/d, hence we have the following:

Given the diameter and the original sagitta; to find the first sagitta.

Put down the original sagitta, divide by 4; this is the original term.

Multiply the original term by the sagitta, divide by the diameter, multiply by 1 and divide by 4; this is the first difference.
Multiply the first difference by the sagitta, divide by the diameter, multiply by 3 and divide by 6; this is the second difference.

Multiply the second difference by the sagitta, divide by the diameter, multiply by 5 and divide by 8; this is the third difference.

Multiply the third difference by the sagitta, divide by the diameter, multiply by 7 and divide by 10; this is the fourth difference.

Multiply the fourth difference by the sagitta, divide by the diameter, multiply by 9 and divide by 12; this is the fifth difference.

Thus multiplying the preceding difference by the sagitta, and dividing by the diameter, and multiplying by the successive odd numbers and dividing by the successive even numbers, we get successive differences, which being added to the original term give us the first sagitta.

In this way, we can easily obtain any number of terms: this is called the h sanitized-to-ono-sho-jitsu (板除得高術). To get the successive terms by the ordinary process would be very troublesome, while by this method it becomes very simple. We must in all such cases carefully consider the matter and by cancelling, putting together, simplifying or substituting, in fact by various means find out a simple law.

The author then proceeds to find the second sagitta (s₂) which is the sagitta of quarter arc.

Put down the second sagitta as unity [the jenge no-achi], subtract it from the diameter and multiply by 4; we get [the sq. of the chord of ¼ arc which is] the approximation to the square of ¼ arc.

The quadratic equation is

\[-ds₂ + 4 (d - s₁) s₂ = 0\]

In this substitute for s₁ from the above. The solution of this equation is given by the following:

\[
\begin{array}{c|c|c|c}
S₀ & S₁ & H₀ & R₀ \\
\hline
\frac{n}{16} & -\frac{d}{8} & -\frac{a_{0}}{8} & \left(-\frac{a_{0}}{8}\right) \\
\frac{5a_{0}^{2}}{16} & -\frac{a_{0}^{2}}{32} & -\frac{a_{0}^{2}}{64} & \left(-\frac{a_{0}^{2}}{64}\right) \\
\end{array}
\]

\[5) \text{ Substituting for } s₁ \text{ from the above we get the series which is printed in thick figures in this column.}\]

&c., &c. [up to 11th term of the result].

We thus get for the second sagitta,

\[
\frac{n}{16} \quad \text{the original term}
\]

\[
+\frac{5a_{1}^{2}}{256d} \quad \text{the first difference}
\]

\[
+\frac{215a_{2}^{2}}{2048d^{2}} \quad \text{" second ..}
\]

\[
+\frac{429a_{3}^{2}}{65536d^{3}} \quad \text{" third ..}
\]

\[
+\frac{243a_{4}^{2}}{524288d^{4}} \quad \text{" 4th ..}
\]

&c. [up to the 10th difference].

The law of the terms is found by the same method as for the first sagitta, viz., put down the original sagitta and divide by 16; this is the original term.
THE LENGTH OF AN ARC OF A CIRCLE.

19. \( s \times 4 \) the 4th difference is the fifth difference.
22. \( s \times 4 \) &c.

Thus we have to multiply the preceding term by two successive odd numbers and divide by two successive even numbers. Moreover the original term of the second sagitta is obtained by multiplying that of the first sagitta by 1 and dividing by 4.

So we may proceed: but for the 3rd, 4th,....sagittae, the process becomes very troublesome, and by considering still more deeply, we shall find that there is a greater law by which we may obtain them at once from the original sagitta. In all cases, the original terms and successive differences depend upon the respective \( jitsu \) (the constant terms) and \( ho \) (the coefficients of the first powers). The respective \( jitsu \) are derived from the one immediately preceding by certain steps. We shall thus see that we may arrive at the result in the following manner.

To find the original terms of the sagittae.

The original term for the 1st sagitta is the original sagitta divided by 4, i.e.

\[
\frac{s}{4}
\]

The original term for the 2nd sagitta has the same numerator but we have to multiply the denominator by 4, so that it

\[
\frac{s}{16}
\]

That for the 3rd sagitta is in the same way

\[
\frac{s}{64}
\]

So for the 4th, &c. sagittae.

To find the 1st difference.

For the 1st sagitta, we multiply the original term by \( s \) and divide by \( d \), multiply by 1 and divide by 4; thus

the 1st difference for the 1st sagitta

\[
\frac{s^3}{16, d}
\]

For the 2nd sagitta, we have for the numerator to add 4 times the numerator of the 1st difference of the 1st sagitta to the square of the numerator of the original term of the second sagitta. To get the denominator we multiply that for the 1st sagitta by 16; thus

\[
\frac{5 \cdot s^3}{256 \cdot d}
\]

For the 3rd sagitta, we have, for the numerator, to add 4 times the numerator of the 1st difference of the 2nd sagitta to the square of the numerator of the original term of the 3rd sagitta. For the denominator, we multiply the corresponding denominator for the 2nd sagitta by 16; thus,

the 1st difference for the 3rd sagitta

\[
\frac{21 \cdot s^3}{4096 \cdot d}
\]

Similarly for the 4th, ....sagittae.

To find the 2nd difference.

For the 1st sagitta, put down the 1st difference, multiply by \( s \) and divide by \( d \), multiply by 3 and divide by 6; we thus get

the 2nd difference for the 1st sagitta

\[
\frac{s^3}{32 \cdot d^2}
\]

For the 2nd sagitta, we have as numerator the sum of 16 times the numerator of the 2nd difference of the 1st sagitta and the product of the numerators of the original term and 1st difference of the 2nd sagitta; as denominator 64 times the denominator of the 2nd difference of the 1st sagitta: thus

the 2nd difference for the 2nd sagitta

\[
\frac{21 \cdot s^3}{2048 \cdot d^2}
\]

For the 3rd sagitta, in the same way, we have as numerator the sum of 16 times the numerator of the 2nd difference of the second sagitta and the product of the numerators of the original term and the first difference of the third sagitta; as denominator 64 times the denominator of the 2nd difference of the 2nd sagitta: thus

the 2nd difference for the 3rd sagitta

\[
\frac{337 \cdot s^3}{131072 \cdot d^3}
\]

Similarly for the 4th, ....sagittae.

To find the 3rd difference.

For the 1st sagitta, multiply the second difference by \( s \), divide by \( d \), multiply by 5 and divide by 8: thus

the 3rd difference for the 1st sagitta

\[
\frac{5 \cdot s^3}{2304 \cdot d^3}
\]

For the 2nd sagitta; as numerator, take the sum of 64 times the
THE LENGTH OF AN ARC OF A CIRCLE.

numerator of the 3rd difference of the 1st sagitta, 4 times the product of the numerators of the original term and the 2nd difference of the 2nd sagitta and once the square of the numerator of the 1st difference of the second sagitta; as denominator 256 times of the 1st sagitta; thus,

the 3rd difference for the 2nd sagitta \[ = \frac{429.5^4}{69536 \text{ d}^4} \]

For the 3rd sagitta; as numerator, take the sum of 64 times that of the 3rd difference of the 2nd sagitta, 4 times the product of the numerators of the original term and the 2nd difference of the 3rd sagitta and once the square of the numerator of the 1st difference of the 3rd sagitta; as denominator 256 that of the 2nd sagitta; thus,

the 3rd difference for the 3rd sagitta \[ = \frac{29325.5^4}{167716 \text{ d}^4} \]

Similarly for the 4th,......sagittae.

To find the 4th difference.

For the 1st sagitta; multiply the 3rd difference by \( s \), divide by \( d \), multiply by 7 and divide by 10; thus,

the 4th difference for the 1st sagitta \[ = \frac{7.5^4}{512 \text{ d}^4} \]

For the 2nd sagitta; as numerator, take the sum of 256 times that of the first sagitta, once the product of the numerators of the original term and the 3rd difference of the 2nd sagitta and twice the product of the numerators of the first and 2nd differences of the 2nd sagitta; as denominator 1024 times that of the 1st sagitta; thus,

the 4th difference for the 2nd sagitta \[ = \frac{2431.5^4}{524288 \text{ d}^4} \]

Similarly for the 3rd, 4th,......sagittae [in the original 4th difference for the 3rd sagitta is given, but I have omitted it here.]

To find the 5th difference.

For the 1st sagitta; multiply the 4th difference by \( s \), divide by \( d \), multiply by 9 and divide by 12; thus,

the 5th difference for the 1st sagitta \[ = \frac{21.5^4}{3048 \text{ d}^4} \]

SEKI'S METHOD OF FINDING

For the 2nd sagitta; as numerator, take the sum of 1024 times that of the 1st sagitta, twice the product of the numerators of the original term and the 4th difference of the 2nd sagitta, once the product of the numerators of the 1st and 3rd differences of the same, and twice the square of the numerator of the 2nd difference of the same; as denominator 4096 times that of the 1st sagitta; thus,

5th difference for the 2nd sagitta \[ = \frac{29393.5^4}{8388608 \text{ d}^4} \]

Similarly, for the 3rd, 4th,......sagittae.

So the author goes on as far as the 7th difference, and then he says, 8th and higher differences can be found in the same way, remembering that the product of the shō and hō is to be put in the jitsu column. The above is the easy way of finding the sagittae without the trouble of the actual solution of quadratic equations, but in order to test the above law as he says, he gives the diagrams of the solution of the eqns for the 3rd and 4th sagittae.

I give here the diagram for the 3rd sagitta in Japanese and English.

Given the diameter and the second sagitta: to find the third sagitta (that of \( \frac{1}{6} \) arc): the solution of the quadratic equation (the method of obtaining the eqn. being the same as before is omitted).
The length of an arc of a circle

In this manner we find the first 5 or 6 terms of the series for the sagittae up to the 10th or so; multiplying them by the diameter and the square of the number of bisection, we get successive approximations to the square of half the original arc. Of course, we are not limited to the tenth, we can proceed to ten-thousand-times-ten-thousandth sagitta and get approximation to the square of the half arc, but here again, as stated before, there is a general law, to obtain which we need not go so very far.

The expressions for the successive sagittae are as in the following table (in the original the differences are given as far as the 5th, and the sagittae as far as the 10th, but I have omitted the 5th difference, and all below the 6th sagittae).

<table>
<thead>
<tr>
<th>The original term</th>
<th>1st difference</th>
<th>2nd difference</th>
<th>3rd difference</th>
<th>4th difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>The original sagitta</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The 1st sagitta (or of half arc)</td>
<td>1/4</td>
<td>1/16 D</td>
<td>1/32 D</td>
<td>1/64 D</td>
</tr>
<tr>
<td>The 2nd sagitta (or of 1 arc)</td>
<td>1/10</td>
<td>1/256 D</td>
<td>1/512 D</td>
<td>1/1024 D</td>
</tr>
<tr>
<td>The 3rd sagitta (or of 1/2 arc)</td>
<td>1/8</td>
<td>1/2048 D</td>
<td>1/4096 D</td>
<td>1/8192 D</td>
</tr>
<tr>
<td>The 4th sagitta (or of 1/4 arc)</td>
<td>1/64</td>
<td>1/16384 D</td>
<td>1/32768 D</td>
<td>1/65536 D</td>
</tr>
<tr>
<td>The 5th sagitta (or of 1/8 arc)</td>
<td>1/256</td>
<td>1/65536 D</td>
<td>1/131072 D</td>
<td>1/262144 D</td>
</tr>
</tbody>
</table>

To obtain approximations to the square of the half arc:

Put down the original sagitta, multiply by the diameter; this is the square of the chord of the half arc, which is the 1st approximation to the square of the half arc. Next put down the 1st sagitta, multiply by the diameter and by the square of the times of bisection (27), we get the 2nd approximation. And so on; thus:
### Seki's Method of Finding

<table>
<thead>
<tr>
<th>Order of approximation</th>
<th>Original Term</th>
<th>1st difference</th>
<th>2nd difference</th>
<th>3rd difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>s, d</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>s, d + (\frac{s^2}{4}) + (\frac{s^3}{8,d}) + (\frac{5,s^4}{64,d^2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>s, d + (\frac{5,s^2}{16}) + (\frac{21,s^3}{128,d}) + (\frac{429,s^4}{4096,d^2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>s, d + (\frac{21,s^2}{64}) + (\frac{357,s^3}{2048,d}) + (\frac{29325,s^4}{262144,d^2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>s, d + (\frac{81,s^2}{256}) + (\frac{5797,s^3}{524288,d}) + (\frac{1907213,s^4}{16777216,d^2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td>s, d + (\frac{341,s^2}{1024}) + (\frac{93093,s^3}{524288,d}) + (\frac{122550285,s^4}{1073741824,d^2})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

&c., up to the 11th approximation in the original.

With the results thus obtained, we seek the limits of the original term and the successive differences. By the limit is meant what we shall ultimately get when [by increasing the times of bisect] the chords protrude out as in stretching a bow; gradually lose their angularities, and finally become an arc of the circle. This is the essential point upon which hangs the whole theory of the circular arc.

*To seek the limits of the original term and the several differences.*

Divide the numerical coefficients of the numerators by those of the respective denominators; we get the following:

<table>
<thead>
<tr>
<th>Approx.</th>
<th>The original term</th>
<th>The 1st diff.</th>
<th>The 2nd diff.</th>
<th>The 3rd diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>1.25</td>
<td>.125</td>
<td></td>
<td>.078125</td>
</tr>
<tr>
<td>3rd</td>
<td>1.3125</td>
<td>.1640625</td>
<td></td>
<td>.1118659973144...</td>
</tr>
<tr>
<td>4th</td>
<td>1.3303135</td>
<td>.17431640625</td>
<td></td>
<td>.113678753376...</td>
</tr>
<tr>
<td>5th</td>
<td>1.3330078125</td>
<td>.1769104000390625</td>
<td></td>
<td>.114133846946060...</td>
</tr>
<tr>
<td>6th</td>
<td>1.333251953125</td>
<td>.1775638063744140625</td>
<td></td>
<td>.114247739504207...</td>
</tr>
</tbody>
</table>

&c. to the 11th approx., each up to 5th difference.

The original term is always 1, so that is the limit of the original term.

In the first difference, we note the figure 3 increases as we proceed to higher approximations; thus we get .33333... (In Japanese we have...}
THE LENGTH OF AN ARC OF A CIRCLE.

special names for decimals, thus $bu=\frac{1}{10}th, rin=\frac{1}{100}th, wot=\frac{1}{1000}th$ &c., so that the above is put down as $3~bu~33333...$] According to the rules of $rei-yaku-justsu$ 6) (箭筈術), we multiply $3~bu$ by the fixed factor $9$, and get $2~bu~7~rin$, add to this $3~rin$, we have $3~bu$ which divided by the fixed divisor $9$, gives us $\frac{1}{9}th$ as the limiting value of the first difference.

In the 2nd differences, similarly, we get gradually figure 7 increase, so that we have $1~bu~777...$; as before, multiply $1~bu$ by $9$, and we have $9~rin$, add $7~rin$, we get $1~bu~6~rin$, which divided by $9$ gives us $\frac{1}{9}th$ as the limit of the 2nd difference.

In the 3rd differences, we find that if we multiply by $7$, in the products so obtained the figure 9 increases, viz, we get nearer and nearer $7~bu~999...$; multiply $7~bu$ by $9$, we have $6~bu~3~rin$; add $9~rin$ and we get $7~bu~2~rin$, divide by $9$ and again by $7$; we get finally $\frac{1}{9}th$ as the limit of the 3rd difference.

In the 4th differences, we find similarly that if we multiply by 7, the figure 8 increases, viz, we get nearer and nearer to $5~bu~6888...$; multiply $5~bu~6~rin$ by $9$, we have $5~bu~04$; add $8~rin$, we have $5~bu~12$, which divided by $9$ and $7$ gives us $128/1575th$ as the limit of the 4th difference.

And so on.

Thus the limits of the original term and the successive differences are:

<table>
<thead>
<tr>
<th>Term</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Term</td>
<td>$sd$</td>
</tr>
<tr>
<td>1st Difference</td>
<td>$1.~s^3/3$</td>
</tr>
<tr>
<td>2nd Difference</td>
<td>$8.~s^3/45.~d$</td>
</tr>
<tr>
<td>3rd Difference</td>
<td>$4.~s^3/35.~d^3$</td>
</tr>
<tr>
<td>4th Difference</td>
<td>$128.~s^3/1575.~d^3$</td>
</tr>
</tbody>
</table>

Dividing each difference by the preceding we get:

<table>
<thead>
<tr>
<th>Term</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Term</td>
<td>$sd$</td>
</tr>
<tr>
<td>1st Difference</td>
<td>$1.~s^3/3.~d$</td>
</tr>
<tr>
<td>2nd Difference</td>
<td>$24.~s^3/45.~d$</td>
</tr>
<tr>
<td>3rd Difference</td>
<td>$180.~s^3/280.~d$</td>
</tr>
<tr>
<td>4th Difference</td>
<td>$4480.~s^3/6300.~d$</td>
</tr>
</tbody>
</table>

---

6) Of this, I may write on another occasion.

SEKI'S METHOD OF FINDING

<table>
<thead>
<tr>
<th>Term</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Term</td>
<td>$sd$</td>
</tr>
<tr>
<td>1st Difference</td>
<td>$4.~s^3/3.~d$</td>
</tr>
<tr>
<td>2nd Difference</td>
<td>$2.~s^3/5.~d$</td>
</tr>
<tr>
<td>3rd Difference</td>
<td>$3.~s^3/7.~d$</td>
</tr>
<tr>
<td>4th Difference</td>
<td>$4.~s^3/9.~d$</td>
</tr>
</tbody>
</table>

Or

<table>
<thead>
<tr>
<th>Term</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Term</td>
<td>$sd$</td>
</tr>
<tr>
<td>1st Difference</td>
<td>$2^2.~s^3/3.~d$</td>
</tr>
<tr>
<td>2nd Difference</td>
<td>$4^2.~s^3/5.~d$</td>
</tr>
<tr>
<td>3rd Difference</td>
<td>$6^2.~s^3/7.~d$</td>
</tr>
<tr>
<td>4th Difference</td>
<td>$8^2.~s^3/9.~d$</td>
</tr>
</tbody>
</table>

Thus then we have to multiply successively by the squares of $2, 4, 6, 8...$ and divide by the products of two successive integers, $3, 4, 5, 6, 7, 8, 9, 10...$

The author then proposes the following problem:

Given the diameter of the circle $= 1~shaku$ (or $10~sui$), sagitta of the arc $= 2~sun$; required to find the length of the arc.

The arc $=$ [not given]

The rule is as follows: multiply the sagitta (s) by the diameter (d) and by 4; the product is the original term. Multiply this by s and by 4, divide by d and by 3,4, this is the second difference. Multiply this by s and by 16, divide by d and by 5,6; this is the second difference. Multiply this by s and by 36, divide by d and by 7,8; this is the 3rd difference. Multiply this by s and by 64, divide by d and by 9,10; this is the 4th difference. Multiply this by s and by 160, and divide by d and by 11,12; this is the 5th difference. Multiply this by s and by 144; divide by d and by 13,14; this is the 6th difference. And so on.
THE LENGTH OF AN ARC OF A CIRCLE.

It might seem that this rule is given as the result of the above; instead of which manuscript gives the process in which we are referred back to the expressions for the saggittae whence the rule is deduced de novo in a somewhat different way from what is given above. In fact, this latter part of the manuscript is very likely to have been a later addition or part of another work. The actual calculation for the above particular case is not given in the manuscript.

The series for successive saggittae obtained above are as follows, as far as the 5th term.

\[
\begin{align*}
S_1 &= \frac{1. s}{4} + \frac{1. s^2}{16. d} + \frac{1. s^3}{32. d^2} + \frac{5. s^4}{256. d^3} + \frac{7. s^5}{512. d^4} \\
S_2 &= \frac{1. s}{16} + \frac{5. s^2}{256. d} + \frac{21. s^3}{2048. d^2} + \frac{429. s^4}{65536. d^3} + \frac{2431. s^5}{524288. d^4} \\
S_3 &= \frac{1. s}{64} + \frac{21. s^2}{4096. d} + \frac{357. s^3}{131072. d^2} + \frac{20325. s^4}{16777216. d^3} + \frac{66655. s^5}{536870912. d^4} \\
S_4 &= \frac{1. s}{256} + \frac{55. s^2}{65536. d} + \frac{5797. s^3}{8388608. d^2} + \frac{1907213. s^4}{4294967296. d^3} + \frac{1735556383. s^5}{549755813888. d^4} \\
S_5 &= \frac{1. s}{1024} + \frac{341. s^2}{1048576. d} + \frac{93093. s^3}{536870936. d^2} + \frac{12250285. s^4}{109951162776. d^3} + \frac{445614732379. s^5}{56294995344312. d^4} \\
\end{align*}
\]

Multiplying \( s_1 \) by \( d \), we get the square of the chord of 1/4 arc: viz,

\[
\begin{align*}
dS_1 &= \frac{1. sd}{4} + \frac{1. s^2}{16} + \frac{1. s^3}{32. d} + \frac{5. s^4}{256. d^2} + \frac{7. s^5}{512. d^3} \\
\end{align*}
\]

Dividing each term by the preceding, we get the factors to be multiplied into the preceding terms in order to get successive terms: viz,

the original term = \( 1. sd/4 = 1. sd/4 \)

the 1st quotient = \( 1. s/4. d = 1.3. s/3.4 d \)

the 2nd term = \( 1. s/2. d = 3.5. s/5.6 d \)

the 3rd term = \( 5. s/8. d = 5.7. s/7.8 d \)

the 4th term = \( 7. s/10. d = 7.9. s/9.10 d \)

---

SEKI'S METHOD OF FINDING

Multiplying \( s_2 \) by \( d \), we get the square of the chord of 1/8 arc, and proceeding as before, we get

the original term = \( 1. sd/16 = 1. sd/4.4 = 1. sd/4.4 \)

the 1st quotient = \( 5. s/16. d = 5. s/4.4. d = 3.5. s/3.4.4. d \)

the 2nd term = \( 21. s/5.8 d = 7.3. s/5.4.2 d = 7.9. s/5.6.4 d \)

the 3rd term = \( 429. s/21.32 d = 3.11.13. s/3.7.8.4 d = 11.13. s/7.8.4 d \)

the 4th term = \( 2431. s/429.8 d = 143.17. s/3.2.4.1.43. d = 15.17. s/9.10.4 d \)

Multiplying \( s_3 \) by \( d \), we get the square of the chord of 1/16 arc and proceeding as before, we get

the original term = \( 1. sd/64 = 1. sd/4.16 \)

the 1st quotient = \( 21. s/6.4. d = 7.3. s/4.16. d = 7.9. s/3.4.16. d \)

the 2nd term = \( 357. s/21.32. d = 7.3.17. s/7.3.2.16. d = 15.17. s/5.6.16 d \)

the 3rd term = \( 29325. s/357.128 d = 3.17.23.25. s/3.17.7.8.16. d = 23.25. s/7.8.16 d \)

the 4th term = \( 66655. s/29325.32. d = 5.17.23.31.11. s/5.3.23.7.5.16. d = 31.33. s/9.10.16 d \)

Multiplying \( s_4 \) by \( d \), we get the square of the chord of 1/32 arc, and in the same way as before we have

the original term = \( 1. s/256 = 1. sd/4.64 \)

the 1st quotient = \( 85. s/256. d = 5.17. s/6.64. d = 15.17. s/3.4.64 d \)

the 2nd term = \( 5797. s/85.128. d = 17.31.11. s/5.17.2.64 d = 31.33. s/5.6.64 d \)

the 3rd term = \( 1907213. s/5797.512. d = 11.17.31.47.7.7 / 8.11.17.31.64. d = 47.49. / 7.8.64 d \)

the 4th term = \( 1735556383. s/1907213.128. d = 7.11.17.31.47.7.13 s/2.7.11.17.31.47.64. d = 63.65. s/3.10.64. d \)

Multiplying \( s_5 \) by \( d \), we get the square of the chord of 1/64 arc, and in the same way as before we have

the original term = \( 1. s/1024 = 1. sd/4.256 \)

the 1st quotient = \( 341. s/1024. d = 31.11. s/4.256 d = 31.33. s/3.4.256. d \)

the 2nd term = \( 93093. s/341.512. d = 11.31.21.13. s/11.31.2.256 d = 63.65. s/5.6.256. d \)
THE LENGTH OF AN ARC OF A CIRCLE.

the 3rd quotient = \(122550285.5/3093.2048\). 
\(d = 3.113.31.95.97.8/3.11.3.31.7.8.256.\) 
\(= 31.113.31.95.97.127.43.8/3.11.31.95.97.3.5.2.256 \) 
\(= 127.129.8/9.10.256.\) 

Put them down as below:

<table>
<thead>
<tr>
<th>The square of</th>
<th>The original term</th>
<th>The 1st quotient</th>
<th>The 2nd quotient</th>
<th>The 3rd quotient</th>
<th>The 4th quotient</th>
<th>The 5th quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/4) arc</td>
<td>(4.1)</td>
<td>3.4.4.1.</td>
<td>3.5.8.</td>
<td>3.7.8.1.</td>
<td>6.10.1.</td>
<td>11.12.1.</td>
</tr>
<tr>
<td>(1/8) arc</td>
<td>(4.4)</td>
<td>3.4.4.4.</td>
<td>3.7.4.</td>
<td>3.7.4.1.</td>
<td>6.10.4.</td>
<td>11.12.4.</td>
</tr>
<tr>
<td>(1/16) arc</td>
<td>(4.16)</td>
<td>3.4.4.16.</td>
<td>3.5.16.</td>
<td>3.7.16.</td>
<td>6.10.16.</td>
<td>11.12.16.</td>
</tr>
<tr>
<td>(1/32) arc</td>
<td>(4.64)</td>
<td>3.4.64.</td>
<td>3.5.32.</td>
<td>3.7.32.</td>
<td>6.10.32.</td>
<td>11.12.32.</td>
</tr>
<tr>
<td>(1/64) arc</td>
<td>(4.256)</td>
<td>3.4.256.</td>
<td>3.5.64.</td>
<td>3.7.64.</td>
<td>6.10.64.</td>
<td>11.12.64.</td>
</tr>
</tbody>
</table>

1) The denominators in each column have as a factor the successive terms of the series \(1^2, 2^2, 4^2, 8^2, 16^2, \ldots\) the other factors being 4 in the 1st column, 3, 5, 7, 9, 11, 12, \ldots respectively in the other columns.

2) For the numerators, we have product of two factors, and
   a) in the 1st quotient, we have to add 2, 4, 8, 16, \ldots successively to each of the preceding factors.
   b) in the 2nd quotient, we have to add 4, 8, 16, 32, \ldots
   c) in the 3rd " " " " " " 6, 12, 24, 48, \ldots
   d) in the 4th " " " " " " 8, 16, 32, 64, \ldots
   e) in the 5th " " " " " " 10, 20, 40, 80, \ldots

From these laws, is derived a table for the original terms and quotients up to the 5th, of the arcs down to 1/2048th of the original arc, and then another table is constructed in which in each quotient the result of dividing the numerator by 1, 4, 16, 64, 256 \ldots respectively is substituted; and then

these results are found to approximate to 4, 16, 36, 64, 100 \ldots as limits; so that in the limit multiplying by sq. of the times of bisection, we have for the sq. of half the arc

- the original term = \(sd\)
- the 1st quotient = \(4.8/3.4.\) \(d\)
- the 2nd " " = 16.8/5.6. \(d\)
- the 3rd " " = 36.8/7.8. \(d\)
- the 4th " " = 64.8/9.10. \(d\)
- the 5th " " = 100.8/11.12. \(d\).

The manuscript ends with a note by T. Honda, as follows:

This book is the work of Master Takebe Fukujû; when he wrote his treatise on the Almanac known as Six Volumes of Takebe (建部之六卷狀) he obtained the method of finding the value of \(\pi\) very closely, and the present work was regarded by him as especially secret, and transmitted by him to my Master Imai who found it very valuable. It was then transmitted to me and I have preserved it as a most valuable work.—T. Honda.
解説

菊池大鵬と藤沢利喜太郎の論文について

わが国において、科学史に関する最初のまとまった著述は、道灌利貞の著『大日本数学史』であることはいまさらいうまでもないが、この名著に前後して、菊池大鵬と藤沢利喜太郎は欧文で和算を外国人に紹介した。

『大日本数学史』は明治29年（1896）の出版であるが、日清戦役の終た翌年である。明治維新後、世界の各国と交流の始まったときに、人が日本人だけの力でこんなすばらしい数学を持っていたのだということを誇示したかったのである。

藤沢利喜太郎は和算の紹介をしたのは、世界的な学者が一堂に会したパリで行なわれた第2回国際数学者会議の席上でである。このときの大会は、ヒルベルトが「数学の未来の問題について」と題して、23のいわゆるヒルベルトの問題を提出した講演であったことでも有名である。

その後、三上義夫は、「The Development of Mathematics in China and Japan, Leipzig, 1913」を著し、さらにD. B. Smithと共著で、「A History of Japanese Mathematics, Chicago, 1914」を出版した。この2者によって、外国人は和算の何であるかを知ったといってよい。さらに林鷹一もまた多くの論文を外国に発表して和算を紹介した。

来年、昭和49年は、京都と東京で第14回科学史国際会議が開催される。科学史の分野で、最初に活躍した和算の部門が外人にどのように理解されているかという意味で、最初の論文を復刻することとした。

なお、先年わが国に来日したニーダム教授に質問したところ、ニーダム教授の和算に関する部分の記事は、三上義夫の著書をそのまま引用したという。その意味で、来年来日する外国人が和算についてどの程度の理解を持っているかということを知るための助けになれば幸である。

（下平和夫）

-59-
編集後記

今回は、明治期に先輩が海外に和算を紹介した諸論文を集めて特集を組みました。主な他については解説をお読み頂きたいと思いますが、これらの復刻は下記によってものです。

藤沢利嘉太郎……藤沢博士記念会編集発行
「藤沢博士論文集（下）」（昭和10年） 169～185頁
菊池泰松………『東京数学物理学会記事』
最初の論文から順に着くと次のようになります。

<table>
<thead>
<tr>
<th>卷</th>
<th>年代</th>
<th>頁数</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>明治28年</td>
<td>24～26頁</td>
</tr>
<tr>
<td>8</td>
<td>明治29年</td>
<td>47～53頁</td>
</tr>
<tr>
<td>9</td>
<td>107～110頁</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>114～117頁</td>
<td></td>
</tr>
</tbody>
</table>

ところで、第13回（昭和49年度）総会は下記の要領で開催される予定です。詳細は次号で御報告致しますが、研究発表を御希望の方は、題目と内容（1.2行程度でもよい）を運営委員会まで御一報下さい。（発表時間は1題目につき15分程度）

日時：昭和49年5月26日（日）午前10時～午後4時
場所：東京都新宿区西新宿1丁目7番18号（富士短期大学高等部）
富士短期大学高田記念館
なお、特別講演の講師は目下交渉中です。また、これより2週間後の6月9日（日）に数学史講座を同じ富士短期大学で開催する予定です。これも詳細決定次第お知らせ致します。

（高木 茂男）

数学史研究

通巻 58号 1973年7月～9月
発行所 日本数学会
東京都新宿区下落合1丁目7番18号 （〒161）
電話 東京（03）368－2254
会費 年額2,000円
振替 東京20022番
印刷所 平山印刷社
東京都文京区本郷2丁目20－6
電話 東京（03）813－0594
沖縄の数学

内容

計算機の歴史

計算と計算機の歴史

下平和夫

数学者を中心とした数学の歴史

数学史

数学者やその数学史

平山 雅男

学術を中心とした算の歴史

平田 痛

珠算の歴史

武久男男

珠算の歴史

下平和夫・柴野企明共著

日本の数学の新知識

山崎 痛

東京 實

山田算左衛門明

山形の算額

古代日本の算額

関西算盤

算盤に関する研究文献解説

第1巻～第4巻

A5判、各120頁、紙質印刷、各巻 印刷 400円
SÜGAKUSHI KENKYU
JOURNAL OF HISTORY OF MATHEMATICS, JAPAN

NO. 58
July - September 1973

CONTENTS

Papers for introduction of the Old Japanese Mathematics

Rikitaro Fujisawa : Note on the Mathematics of the
Old Japanese School. ........................................ (1)

Dairoku Kikuchi : On the method of the Old Japanese
School for finding the area of
a circle. .......................................................... (18)

" : Various series for π obtained by the
Old Japanese Mathematicians. ............................... (21)

" : A series for π² obtained by the Old
Japanese Mathematicians. ................................... (28)

" : Ajima's method of finding the length
of an arc of a circle. ......................................... (32)

" : Seki's method of finding the length
of an arc of a circle. ........................................ (36)

Edited and Published by
The History of Mathematics Society of Japan
Fuji Junior College
1-7-18, Shimooshiai, Shinjuku-ku, Tokyo, Japan